Inertia in the Structure of Four-dimensional Space

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Abstract

1. Following Rimman, Minkowski and Einstein, for the first time equations of the inert filed in the covariant form are found geometrically.

2. In the approximation of a weak field for the first time the Law of Inertia in a material space (as opposed to the absolute space) is received. A consequence is the formulation of Mach's principle.

3. Analogous to Einstein's expression for the gravitational field $c = c_0 \left(1 - \frac{2GM}{r}\right)$, *Compton's formula is received for the inert field* $r = \frac{h}{mc}(1 - l_0 \cos\theta)$.

4. For the first time transcendental equations are received, one of the solutions of which corresponds to the value of the magnetic charge by Dirack, or to the constant of fine structure.

Minkowski's work [1] for the first time led the solution of physical problems to geometrical problems.

1. Such physical theorem as kinematics and theory of inertial systems is obtained on the basis of flat space through equation

$$R_{iklm} = 0 \tag{1}$$

for three-dimensional Euclidean as well as for four-dimensional pseudo-Euclidean spaces.

2.

$$G_{ik} = R_{ik} - \frac{1}{2} R g_{ik} \tag{2}$$

is presented as an object of four-dimensional pseudo-Rimman geometry (Einstein) and is determined by formula

$$K = \frac{G_{ik}\xi^i \xi^k}{\xi^i \xi_i} \tag{3}$$

where K is the mean scalar crookedness of three-dimensional subspace, which is orthogonal to an arbitrarily determined vector ξ^i. Tensor G_{ik} is called the conservative tensor of Gilbert-Einstein and plays a fundamental role in the structure of the field equation of the General Theory of Relativity (GTR).

Let us discuss formula

$$G_{ik} = 0 \tag{4}$$

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Einstein [2] predicted that this equation might be a basis for a more general physical theory in case it described electromagnetic fields along with gravitational fields. It is true that this equation is insufficient for describing all physical fields. It describes only gravitational fields outside of the source. From the point of view of geometry this equation is a necessary but not sufficient condition for flat space.

For a precise description of the flat space along with equation (4) we will try to introduce one more geometric equation. The starting point of the unified field theory[2] is the fact that equation (4) alone is insufficient for describing all phenomena. Therefore the problem of finding a mathematical object occurs which, after being added to G_{ik} , will allow to obtain the structure of space describing the physical situation completely.

Gravitation can be described either by introducing the gravitational field into the flat space, or by discussing a geometrical object in crooked four-dimensional space without the gravitational field. The last simulation is the geometric interpretation of gravitation. Hence if the deformation, which creates crookedness is excluded, we will obtain the flat space.

Mathematical Part

Let us introduce tensor $\overline{G}_{ik}$, P_{ik} in the following way:

$$G_{ik} + \frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i} = P_{ik}$$

$$\overline{G}_{ik} - \frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i} = P_{ik} \tag{5}$$

$\overline{G}_{ik}$ is the counter-tensor of G_{ik} , which describes the counter-space in relation to space determined by G_{ik}. η_{ik}, g_{ik} are metric tensors of flat and crooked spaces respectively, and ξ^i is the vector determined initially.

Geometrically (5) means that two spaces of Einstein $\overline{G}$ and G are called mutually counter if the following is realized: In each point of space the ratio of mean scalar crookednesses of the three-dimensional subspace orthogonal to initially determined vector ξ^i depends on the module of that vector only. Both spaces have a common coordination.

$$\frac{K}{\overline{\overline{K}}} = \frac{e_i e^i}{\xi^i \xi_i} \tag{6}$$

where e_i is the unit vector of vector $\xi_{i.}$

The mean scalar crookedness of the three-dimensional subspace is determined by formula (3) and therefore (6) is written as

$$\xi^i \xi_i G_{ik} \xi^i \xi^k = e_i e^i \overline{G}_{ik} \xi^i \xi^k \tag{7}$$

Hence we can determine

[2] Inert field is included with the fieled of gravitation

$$G_{ik} - 2\frac{g_{ik}}{\xi^i \xi_i} = \overline{G}_{ik} - 2\frac{\eta_{ik}}{\xi^i \xi_i}$$

$$G_{ik} + \frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i} = \overline{G}_{ik} - \frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i} \tag{8}$$

Let us introduce operator $(-)^n$ which transforms space into counter-space, tensor G_{ik} into tensor $\overline{G}_{ik}$, and the sign is changed into the opposite one. Even times of counter-operating transforms the space into itself, and odd times of counter-operating transforms the space into the counter-space. If the left side is noted as tensor P_{ik} and the right side is noted as counter-tensor $(-1)\,P_{ik}$, then equation (8) can be written as

$$P_{ik} = (-1)\,P_{ik} \tag{9}$$

Theorem 1

For the space to be flat it is necessary and sufficient that tensor G_{ik} and counter-tensor $\overline{G}_{ik}$ be equal.

Proof - necessity: From equation system (5) we receive

$$G_{ik} - \overline{G}_{ik} - 2\frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i} = 0 \tag{10}$$

Sufficiency: If $G_{ik} - \overline{G}_{ik} = 0$ is a flat space and vice versa $\eta_{ik} - g_{ik} = 0$, then $G_{ik} = \overline{G}_{ik}$. Theorem proved.

From system (5) follows equation

$$G_{ik} + \overline{G}_{ik} = 2P_{ik} \tag{11}$$

If $\overline{G}_{ik} = 0$, then we receive the already familiar structure of Einstein equation [3], only if we substitute $2P_{ik}$ by tensor of energy of impulse and tension.

Theorem 2

For the space to be flat, it is necessary and sufficient that $P_{ik} = 0$.

Equation (5) is written as

$$G_{ik} + \frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i} = 0 \tag{12}$$

Based on the example of spherical symmetry, equation (12) has a flat solution (metrics of Minkowski).

Consequence: For flat space we have

$$G_{ik} - \overline{G}_{ik} = 0$$

$$G_{ik} + \overline{G}_{ik} = 0$$

Hence, for the space to be flat, it is necessary and sufficient that

$$G_{ik} = 0, \overline{G}_{ik} = 0 \tag{13}$$

or that space and counter-space coincide and be equal to zero.

Thus the condition of sufficiency of $\overline{G}_{ik} = 0$ is added to the condition of necessity of flat space $G_{ik} = 0$.

If we look at equation (11) from the mathematical point of view, then tensor $2P_{ik}$ is a source of crookedness or a source of deformation.

However!.. Expression (10) can be decomposed by two menas into two equivalent equations:

$$G_{ik} = 2\frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i} \qquad \overline{G}_{ik} = 0 \qquad \text{(a)} \qquad \text{(first way)}$$

$$\overline{G}_{ik} = -2\frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i} \qquad G_{ik} = 0 \qquad \text{(b)} \qquad \text{(second way)} \tag{14}$$

Physical Part

The first pair of equations (14,a) describes the internal problem of gravitation, or the external problem of inertia, and the second pair of equations (14,b) - vice versa. Based on the above, the full problem of gravitation can be formulated:

$$G_{ik} = 0$$

$$G_{ik} = 2\frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i} \tag{15}$$

where the first equation is the external problem, and the second equation is the internal problem. In the right side of the second equation of system (15), the tensor of energy, impulse and tension is substituted by $T_{ik} = 2\frac{(\eta_{ik} - g_{ik})}{\xi^i \xi_i}$ - a tensor of geometric origin (magnitude of deformation of space). The solution of system (15) will be searched for spherical symmetry or for a centrally symmetric field.

The solution of the first equation (15) is the famous solution of Schwarzschild [4] and is expressed by metrics:

$$ds^2 = \left(1 - \frac{r_{gr}}{r}\right) c^2 dt^2 - r^2 \left(\sin^2\theta d\varphi^2 + d\theta^2\right) - \frac{dr^2}{1 - r_{gr}/r} \tag{16}$$

and the second equation of system (15), which describes gravitational filed inside the source, is expressed by the system of equations (is calculated by components of the tensor):

$$\exp(-\lambda)\left(\frac{\partial\lambda}{\partial r}\frac{1}{r} - \frac{1}{r^2}\right) - \frac{1}{r^2} = 0$$

$$\frac{\partial \lambda}{\partial t} = 0 \tag{17}$$

where

$$g_{00} = \exp(-\lambda)$$

$$g_{33} = \exp(\lambda)$$

the solution of which is presented by metrics

$$ds^2 = \left(1 - \frac{r}{r_k}\right) c^2 dt^2 - r^2 \left(\sin^2 \theta d\varphi^2 + d\theta^2\right) - \frac{dr^2}{1 - r/r_k} \tag{18}$$

For small distances it becomes the metrics of usual pseudo-Euclidean geometry. Another constant is determined from the condition of coincidence on the border of the internal and the external solutions. If in the solution of Schwarzschild that constant is expressed as gravitational radius and is a geometrical measure of the active gravitational mass, then for (18) that constant represents the inert radius and expresses the geometrical measure of the active inert mass. From the condition of coincidence on the border of the internal and the external solutions, we have

$$1 - \frac{r_{gr}}{r} = 1 - \frac{r}{r_k}, \text{ or } \quad r_{gr} r_k = r^2 = r_0^2, \quad \frac{Gm_{gr}}{c} r_k = r_0^2$$

If constant r_0 is the Plank distance (expressed through natural constants), then for r_k we receive expression

$$r_k = \frac{\hbar}{mc}$$

Mass is introduced into physics with different notions: First as inert resistance, inert passive mass m_{PI}, and secondly, as a constant of association which shows how strongly the gravitational field φ affects the body. This is a constant of association, which is the passive gravitational mass m_{PG} (as the mass of a test body). And finally, the third notion is the active gravitational mass m_{AG}, as gravitational charge or as intensity of the source of gravitational field. The first two notions are included in the equation of motion:

$$m_{PI} \frac{d^2 x^i}{dt^2} = m_{PG} \frac{d\varphi}{dx^i}$$

The third notion is included in the gravitational potential. The principle of equivalence assumes the universal law

$$m_{in} = m_{gr,} \qquad m_{PI} = m_{PG} \tag{c}$$

Note that in formulas where metric coefficients represent potentials of specific force fields, gravitational radius is included, which is determined by the active gravitational mass (m_{AG}). In the solution of Schwarzschild these are potentials of gravitational filed. Metrical coefficients of the counter-space are interpreted as potentials of the inert field.

Mass m_{AI}, included in the Compton radius, is the fourth notion of mass. It is the center of dispersion of light and is included in the structure of metrical tensor of the counter-space $\overline{g}_{ik}$. The fourth notion of mass is presented as the active inert mass. This was noted by Siama. [5].

The equality between the active gravitational and the active inert masses, which are present in formulas (16) and (18) respectively, is formulated as the active principle of equivalence.

$$m_{gr} = m_{in}$$

and as a consequence $r_0 = \dfrac{(Gh)^{1/2}}{c^3}$ is the formula expressing the fundamental meaning of length, only if $r_{gr} = r_{in} = r_0$.

From the condition of coincidence on the border of the internal and the external solutions, (16) and (18) coincide in point r_0 and are expressed as

$$ds^2 = \left(1 - \frac{m}{m_0}\right) c^2 dt^2 - r^2 \left(\sin^2 \theta d\varphi^2 + d\theta^2\right) - \frac{dr^2}{1 - m/m_0} \qquad (19)$$

where $m_0 = \left(\dfrac{\hbar c}{G}\right)^{1/2}$ is the fundamental meaning of mass.

Thus if the metrics of Schwarzschild describes the external gravitational filed with source r_{gr}, then metrics (18) describes the external inert filed with source r_k.

Gravitational and inert fields are linked through the active principle of equivalence in the counter-projection. Let us consider particular cases.

1. Approximation of a weak field

The expression of component g_{00} of the metric tensor

$$g_{00} = 1 - \frac{r}{r_k}$$

describes the potential of the filed, and the force is determined by formula

$$F = mc^2 \frac{dg_{00}}{dr}, \qquad F = \frac{mc^2}{\hbar/Mc} = \frac{Mmc^3}{\hbar} \frac{\vec{r}_0}{|r_0|} \qquad (20)$$

where M is the active inert mass, m is the mass of the test body, and $\vec{r}_0$ is the unit vector. In formula (20) force F does not depend on the distance, which means that in the formation of that force all components and fragments of the Universe participate equally. And if the Universe is homogeneous and isotropic then due to $M = 0$ (effective mass), this force $F = 0$. Newton's first law is obtained - the theory of inertial systems. By the way, in this approximation from Schwarzschild's solution the Law of Terrestrial Gravitation is obtained.

The forces of inertia are so usual that it is very uncommon to think about such a problem. Despite this, a lot has been written and spoken about this. Here we would like to discuss sources of inertial forces. According to Newton [6], the source of inertial forces is the absolute space. This means that the absolute space can influence matter, and the opposite influence form the matter is excluded. Thus, the interaction between the absolute space and material bodies does not exist. This rather abstract point of view corresponds to the interpretation of physical interactions on the basis of the field theory. Therefore, a result of the proposed theory is that forces of inertia impacting on the body depend on physical qualities of space and all members filling the space.

Basically, Mach's principle is formulated: Inert qualities of an object are determined by distribution of mass-energy in the whole space. On the basis of Mach's principle and the active principle of equivalence, as well as on the basis of Rimman's idea on geometry of space corresponding to physics and playing an essential role in it, we were able to build this theory.

Rotation of a body relative to the system of static stars is equivalent to the rotation of stars around the body. In both cases the relative motion is the same. This is relativity according to Mach and Berkley.

Let us imagine a body located inside of a massive sphere filled with members of Metagalaxy. If the body suddenly gets acceleration, then such acceleration will be similar to the acceleration of the whole Universe relative to the body. In this process all members of the Universe participate equally regardless of their distance. If the Universe is homogeneous and isotropic, then such acceleration for the body means distortion of local isotropness (rotation) or local homogeneity (uniform motion). Forces of inertia occur. Mach proposed a viewpoint similar to this ideology. A system is examined in the Universe [7], in which a big amount of matter exists on far distances.

Centrifugal forces occur due to real rotation around a real rotation axis, and thus the local isotropness is distorted in the system of the Universe. Forces of Coriolis occurring in a rotating system of coordinates are real forces and are created by the rotation of the whole Universe around the considered body.

2. Let us discuss a particular case when the metrics is given on a three-dimensional sphere and is determined by formula:

$$ds^2 = r^2 \left(\sin^2 \theta d\varphi^2 + d\theta^2\right) \tag{21}$$

That is equivalent to the motion of a three-dimensional spherical surface in the radial direction in the four-dimensional pseudo-Rimman space (18).

$$\left(1 - \frac{r}{r_k}\right) c^2 dt^2 - \frac{dr^2}{1 - r/r_k} = 0$$

or

$$r = r_k \left(1 - \cos\theta\right), \qquad r = \frac{h}{mc} \left(1 - \cos\theta\right) \tag{22}$$

Compton's formula is received for dispersion of light on charged particles if $\frac{v}{c} = \cos\theta$, $r_k = \frac{h}{mc}$ are the Compton or inertial radiuses. In Schwarzschild's solution this particular case brings us to Einstein's formula [8], which was obtained before the creation of GTR.

$$c = c_0 \left(1 - \frac{2GM}{c^2 r}\right) \tag{23}$$

The observed Compton effect on charged particles is an expression of the active inert mass. For big m this effect is not observed due to its small value.

In the metrics of Schwarzschild when $r \to r_{gr}$ all processes on the body in relation to the external observer are "frozen", and a collapse occurs which creates a frozen body, which does not send any signals to the surrounding environment and interacts with the external world only through its static gravitational field. Such a formation is called a gravitational black hall, or a gravitational collapse.

In metrics (18) when $r \to r_k$, all processes on the body in relation to the internal observer are frozen. A frozen inert black hall is formed, which interacts only through its inert filed. The source of the inert field is the equivalent mass of energy of gravitational field. Analogously, the source of the gravitational field is the equivalent mass of energy of the inert field.

Suppose $r \to r_{gr}, r_k$, then the body will be within gravitational as well as within inertial radiuses. Such situation will occur only in point $r_0 = \left(\frac{GM}{c^3}\right)^{1/2}$, which is determined by the Plank distance. This point r_0 in the metrics of the four-dimensional pseudo-Rimman space seems to us to be an absolute and special point. A body determined in such a way may be considered both blind and deaf.

If gravitational and inert fields are created by spherical bodies, then their full mass is expressed as

$$m = \frac{4\pi}{c^3} \int P_0^0 r^2 dr \tag{24}$$

For the gravitational field the tensor is determined by component

$$P_0^0 = \frac{2r_0}{r^3} \tag{25}$$

and for the field of inertia

$$\overline{P}_0^0 = \frac{2}{r r_0} \tag{26}$$

determining the limits of integration. The dead particle with radius $r_0 = \left(\frac{GM}{c^3}\right)^{1/2}$ and mass does not create any excitation in the space. The latter remains relativistically flat, although the space is like being filled up with dead mass (blind and deaf particles).

Such virtual particles are foundations, on which real particles are born by localizing equivalent energy in the given volume of space through exciting it. That is why it is assumed that during formation of the particle the limits of integration are changed from 1 to r on the one side, and from 1 to $1/r$ on the other. Thus,

$$m = \int\limits_{1/r}^{r} \frac{2r_0}{r^3} r^2 dr \qquad \text{if} \qquad x = \frac{r_0}{r}$$

for the source of gravitational field

$$m = \int\limits_{1/x}^{x} \frac{dx}{x} = 2\ln\left|x^2\right| \tag{27}$$

for the source of the inert field

$$\overline{m} = \int\limits_{1/x}^{x} x dx = \frac{1}{2}\left(x^2 - \frac{1}{x^2}\right) \tag{28}$$

Using the active principle of equivalence again, we receive

$$x^4 - 1 = 4\left|x^2\right| \ln\left|x^2\right| \tag{29}$$

A transcendental algebraic equation is obtained, the roots of which are

$$|x_0|^2 = 1, \qquad x_1 = x_0 \alpha^{1/4}$$

and their reverse values.

We think that $x_1^4 = j$ is the value of Dirack's monopole, if the electric charge $e^2 = 1$, then $2j^2 = \alpha^2$ is the value of the constant of the fine structure.

To conclude, we would like to mention that physical consequences of this work, including cosmological consequences, are very interesting and will be discussed in a separate article.

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